Conclusions

The application of the finite strip method to supersonic flutter of composite laminated panels has been presented. The present formulations are for symmetric laminates but it is easy to extend the formulations to general laminated plates. Based on the present results, the following conclusions can be made:

1) For isotropic panels, the number of strips and series terms that required giving satisfactory results by the finite strip method is dependent on the flow angularity.

2) When fiber orientation is not aligned with the x- or y-direction, increasing series terms will rapidly improve the accuracy of the results.

3) Flutter boundary (λ_{cr}) is independent of the series terms when the airflow is along the x-direction ($\Lambda = 0°$) and is independent of the strip numbers when the airflow is along y-direction ($\Lambda = 90°$).

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Bounded-Variable Gauss–Newton Algorithm for Aircraft Parameter Estimation

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Introduction

ESTIMATION of stability and control derivatives or of nonlinear unsteady aerodynamic effects from flight data is a subject of continuous interest. The time-domain approach based on the output error method is widely used for this purpose.[1–3] It leads to a nonlinear optimization problem, which is solved mostly using the unconstrained Gauss–Newton method. Parameter estimation subject to simple bounds can, however, be relevant in some cases. Two typical applications are the following: 1) parameters that describe the physical effects, in the present case aerodynamic effects, are often constrained to lie in a certain range, for example, the Oswald's factor[4] characterizing the increase in drag over ideal condition caused by nonelliptical lift distribution and interference is typically limited to less than one or the time delay is always positive and hence greater than zero; and 2) estimation of highly nonlinear model parameters such as friction, which may lead to numerical difficulties caused by different reasons like poor guess of initial values.[5] Incorporation of such lower and upper bounds in aircraft parameter estimation using the Gauss–Newton method has not been hitherto reported in the literature. This Note, therefore, addresses the issues pertaining to extending the Gauss–Newton method to account for simple bounds and also demonstrates that the active-set strategy provides an efficient solution retaining the desirable properties of the Gauss–Newton method, namely, quadratic convergence and availability of statistical information regarding the accuracy of the estimates.

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Problem Formulation

In the general case a dynamic system is represented as

$$\dot{x}(t) = f[x(t), u(t), \lambda] \qquad x(t_0) = x_0 \tag{1}$$

$$y(t) = g[x(t), u(t), \lambda] \tag{2}$$

$$z(t_k) = y(t_k) + v(t_k) \qquad k = 1, 2, 3, \ldots, N \tag{3}$$

where x is the n-dimensional state vector, y the m-dimensional observation vector, and u the p-dimensional control input vector. The system functions f and g are general nonlinear real valued vector functions. The measurement vector z is sampled at N discrete time points t_k, and the noise vector v is assumed to be a sequence of independent Gaussian random variables with zero mean and covariance matrix R. It is required to estimate the unknown system parameters λ and the initial conditions x_0 as well as the measurement noise covariance matrix R.

Unconstrained Gauss–Newton Method

The maximum likelihood estimates of the unknown parameters and of the unknown noise covariance matrix are obtained by minimizing the cost function[3,6]:

$$J(\Theta, R) = \frac{1}{2}\sum_{k=1}^{N}[z(t_k) - y(t_k)]^T R^{-1}[z(t_k) - y(t_k)]^T + \frac{N}{2}\ln|R| \tag{4}$$

where $\Theta = [\lambda^T, x_0^T]^T$ denotes the q-dimensional vector of unknown parameters, which may be extended to include bias errors in the measurements of response and control input variables.[6] Optimization of Eq. (4) is carried out in two steps. In the first step it can be shown that for any given value of Θ the maximum likelihood estimate of R is given by

$$\hat{R} = \frac{1}{N}\sum_{k=1}^{N}[z(t_k) - y(t_k)][z(t_k) - y(t_k)]^T \tag{5}$$

Having obtained an estimate of R, any optimization method can be applied to update the parameter vector Θ. The investigations in the past have, however, demonstrated that the derivative-free search methods such as Powell and downhill Simplex methods[7] or Extrem[8] and routinely available gradient-based methods such as quasi-Newton, conjugate-gradient, or Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithms[7] are much slower compared to the Gauss–Newton method, particularly for estimation involving large dynamic systems where the computational effort to compute the system responses and their gradients is high.[9,10] For aircraft parameter estimation purposes the Gauss–Newton method is therefore widely used.[1–3] The unconstrained Gauss–Newton method yields the iterative parameter update:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \quad \text{with} \quad \Delta\Theta = -F^{-1}G \tag{6}$$

where the $q \times q$ dimensional information matrix F and the q-dimensional gradient vector G are given by

$$F = \frac{\partial^2 J}{\partial \Theta^2} \approx \sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}\left[\frac{\partial y(t_k)}{\partial \Theta}\right] \tag{7}$$

$$G = \frac{\partial J}{\partial \Theta} = \sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}[z(t_k) - y(t_k)] \tag{8}$$

The response gradients $\partial y/\partial\Theta$ required to compute F and G using Eqs. (7) and (8) are approximated using the finite differences.

The maximum likelihood estimation being asymptotically bias free and efficient, the information matrix F provides a good approximation to the parameter error covariance matrix.[1,3] Standard deviations of the estimates and coefficients of correlation among them can be readily obtained because the information matrix is already computed as a part of the optimization.

Bounded-Variable Gauss–Newton Method Using Active Set Strategy

The linearly constrained optimization problem in which the constraints are simple bounds on the variables is formulated as

$$\min_{\Theta} J(\Theta) \quad \text{subject to} \quad \Theta_{\min} \le \Theta \le \Theta_{\max} \tag{9}$$

Optimization theory provides several approaches to solve this problem, such as transformation of variables followed by unconstrained optimization, barrier function, or Lagrangian approach, or active set strategy.[7] Several software programs, for example, limited memory BFGS,[11] Extrem,[8] quasi-Newton,[12] bounded-variable Least Squares,[13] provide solutions to this problem. However, as already pointed out the Gauss–Newton method is preferred here. The active set strategy is conceptually very appealing and can be readily extended to the Gauss–Newton method.

Starting from the initially specified parameter values Θ_0, an active set, denoted as IA, containing the indices of the variables hitting the bounds is formed and updated in every iteration. A variable is called a free variable, if it is within the permissible bounds, and hence not in the active set. The Gauss–Newton search directions for the free variables are computed as follows:

$$\Delta\Theta_{\text{free}} = -F_{\text{free}}^{-1} G_{\text{free}} \tag{10}$$

where the information matrix F_{free} and the gradient vector G_{free} are computed using Eqs. (7) and (8) respectively for the free variables. The parameter updates resulting from Eq. (10) are checked for the specified bounds, and any violation leads to modification of the active set IA. For such parameters the values are set to the respective bounds and the search directions of Eq. (10) to zero. For the remaining free parameters a new point is computed using a line search.

An important aspect of the active set strategy is to appropriately alter the active set IA as the optimization progresses. The active set is changed whenever a free variable hits its bounds during an iteration. Furthermore, if the Kuhn–Tucker optimality conditions

$$(G_i < 0, \quad \text{for} \quad \Theta_i = \Theta_{i\_\max}) \qquad \text{or}$$
$$(G_i > 0, \quad \text{for} \quad \Theta_i = \Theta_{i\_\min}) \tag{11}$$

are not satisfied for any of the variables in the active set, then those variables are dropped from the active set and made free; G_i, Θ_i, $\Theta_{i\_\min}$, and $\Theta_{i\_\max}$ are respectively the components of the gradient vector given by Eq. (8), the current parameter value, and its lower and upper bounds. In other words, Eq. (11) guarantees that the gradients for the variables hitting the bounds are such that they point outward of the feasible region, implying that any further minimization of the cost function would be possible only when the particular parameters are not constrained within the specified limits.

The computational overhead to implement the active set strategy in the existing Gauss–Newton method is minor; it is just required to check for the variables that hit the bounds and for the optimality conditions of Eq. (11) to update the active set. The advantages of the approach are that the optimization method retains the quadratic convergence property and, as already pointed out, the accuracy of the parameter estimates can be readily obtained.

Example

To verify the extension of the Gauss–Newton method to bounded variables and to evaluate its performance, a typical example of estimating lateral-directional derivatives is considered. The flight data analyzed are for the clean configuration of a C-160 Transall military transport aircraft at 16,000 ft (4876 m) and an indicated airspeed of 160 knots.[14] The equations of motion pertaining to the lateral-directional motion are well documented in the literature and hence not presented here; it would suffice to mention that the state variables are $[\beta, p, r, \phi, \psi]$ and the control inputs are aileron and rudder deflections $[\delta_a, \delta_r]$.

Table 1 Progress of the optimization

	Cost function (Number of variables at bounds)	
Iteration	Unconstrained Gauss–Newton method	Bounded-variable Gauss–Newton method
0	5.135D-22	5.135D-22 (0)
1	5.681D-36	1.211D-34 (3)
2	7.097D-39	1.984D-38 (4)
3	5.085D-39	1.416D-38 (3)
4	5.025D-39	1.369D-38 (2)
5	5.023D-39	1.368D-38 (2)
6	5.023D-39	1.368D-38 (2)

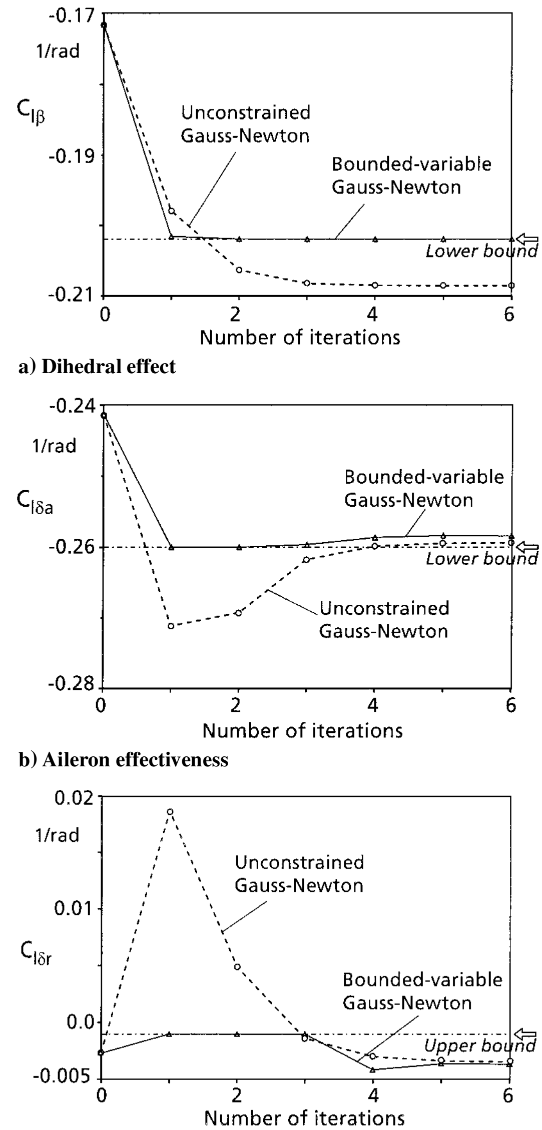
The rolling moment coefficient for this trim point is modeled as

$$C_l = C_{l\beta}\beta + C_{lp}p + C_{lr}r + C_{l\delta a}\delta_a + C_{l\delta r}\delta_r \tag{12}$$

where β is the angle of sideslip, p the roll rate, and r the yaw rate; it is required to estimate the derivatives $[C_{l\beta}, C_{lp}, C_{lr}, C_{l\delta a}, C_{l\delta r}]$ along with those for the yawing moment coefficient C_n and side-force coefficient C_Y. Parameter estimation is carried out applying the conventional unconstrained Gauss–Newton and the bounded-variable Gauss–Newton method presented in this Note. During the bounded-variable Gauss–Newton method, for demonstration purposes the lower and upper limits were specified arbitrarily for four parameters $C_{l\beta}$, $C_{l\delta a}$, $C_{l\delta r}$, and $C_{n\beta}$. The progress of the optimization in terms of cost function is shown in Table 1. The observation is made that for this example the bounded-variable Gauss–Newton method requires the same number of iterations to converge to the minimum. The first iteration step of the bounded-variable method leads to three parameters hitting the bounds and the second iteration to four as evident from the numbers given in the brackets. The third iteration, however, leads to violation of the optimality condition mentioned in the foregoing section for one of the parameters, namely, $C_{l\delta a}$, and hence it is dropped from the active set and made free. The same happens for another parameter $C_{l\delta r}$ during the next iteration, and hence the number of variables in the active set is further reduced to two. For all subsequent iterations the parameters $C_{l\beta}$ and $C_{n\beta}$ remain at their bounds satisfying the optimality condition. Salient features of working of the active set strategy, i.e., the process of variables entering in or dropping out of active set, are brought out in Fig. 1, which provides comparative plots showing the convergence of the estimates and the respective bounds for dihedral effect $C_{l\beta}$, aileron effectiveness $C_{l\delta a}$, and rolling moment caused by rudder deflection $C_{l\delta r}$.

As usually is the case, the unconstrained optimization leads in this case also to a lower minimum compared to the bounded-variable optimization. The differences in converged numerical values of Table 1 may appear to be somewhat large. However, it needs to be remembered that the cost function represents the determinant of the covariance matrix accounting for multiple observation variables and is extremely small in value; in terms of the match between the flight measured and model predicted responses, not shown here, hardly any differences were apparent in the two cases. The estimates of $C_{l\delta a}$ and $C_{l\delta r}$ agreed well within scatter given by the standard deviations of the parameter estimates. The minor differences seen in Figs. 1b and 1c respectively are attributed to the bound optimization.

In the foregoing example and in some other test cases not presented here, the observation was made that the bounded-variable Gauss–Newton method usually required the same number of iterations for the convergence. However, in some test cases it was observed that the convergence was affected, requiring typically one or two additional iterations, particularly if the bounds were inappropriate. Thus, in general, some care is necessary while specifying the parameter bounds. In some cases cycling of active-set variables, i.e., variables entering the active set on reaching the bounds followed



a) Dihedral effect

b) Aileron effectiveness

c) Rolling moment caused by rudder deflection

Fig. 1 Iterative estimates of typical rolling moment derivatives.

immediately by indications to leave the active set, can result, particularly as the minimum of the cost function is approached. The reason for this is twofold: 1) the roundoff errors becoming dominant, and the gradients computed using the numerical approximations are inaccurate; and 2) some variables are approximately linearly dependent. This phenomenon was, however, not encountered in several examples of estimating nonlinear aerodynamic parameters with various degrees of complexity.

Conclusion

The widely used Gauss–Newton method for aircraft parameter estimation in the time domain has been successfully extended to account for simple bounds on the variables. From an engineer's point of view and for implementation purposes, the active set strategy appears to be a simple, direct, and efficient approach. Additionally, the approach retains all of the advantages of the classical unconstrained Gauss–Newton at marginally larger computational overhead. The method extends, in general, the scope of aircraft parameter estimation by permitting the limitation of the variables to be estimated within a specified range. The performance of the bounded-variable Gauss–Newton method presented in this Note was demonstrated on a typical example of estimating the stability and control derivatives pertaining to the lateral-directional motion of an aircraft from flight data.

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Errata

Implications of the Insensitivity of Vortex Lift to Sweep

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EQUATION (6) should be:

$$Cp_v(x, y) = 1 - \left(4.63 \tan^{1.2}\alpha \cos\alpha \left(\frac{2s}{b}\right) \times \cos^2\left\{\tan^{-1}\left(\frac{1}{z_v}\left(\frac{y}{s} - y_v\right)\right)\right\} \Bigg/ 2\pi z_v \tan^{0.2}\epsilon\right)^2$$

On page 532, the second sentence of the last paragraph should read: "This may be considered an upper (or rearward) bound as the wing sweep tends to 90 deg and the wing's trailing-edge extent tends to 0."